

# Focusing Device Based on Overconstrained Mechanism

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**Abstract.** The paper presents an application based on an exceptional 6R overconstrained mechanism. This mechanism is started from 6R Wohlhart symmetric mechanism, not in its classical spatial position but a different one, with three non-adjacent joints constrained to remain in a fixed plane. This disposition allows the conception of an interesting device, a focusing table, capable of obtaining good accuracy.

## 1. Introduction

The paper analyzes a mechanism with six revolute joints in a closed loop, namely Bricard rectangular mechanism or Wohlhart symmetric mechanism. Overconstrained mechanisms with revolute joints are composed of four, five, or six joints. Sarrus [1] presents in 1853 a 6R closed-loop mechanism with six revolute joints, transforming a rotation movement in a translation one. Raoul Bricard described in his papers [2,3] six types of overconstrained linkages, three mobile octahedral mechanisms, and three symmetric mechanisms. J.E. Baker analyze in 1980 these mechanisms [4], giving the closure equations for all these mechanisms. Karl Wohlhart [5] presents in 1987 a new overconstrained mechanism with six revolute, considered as a generalization of Bricard rectangular (or threefold symmetric) mechanism. Other researchers have studied these types of mechanisms, like Bennett [6], Myard [7] or Goldberg [8], Waldron [9], Mavroidis and Roth [10] or Dietmaier [11].

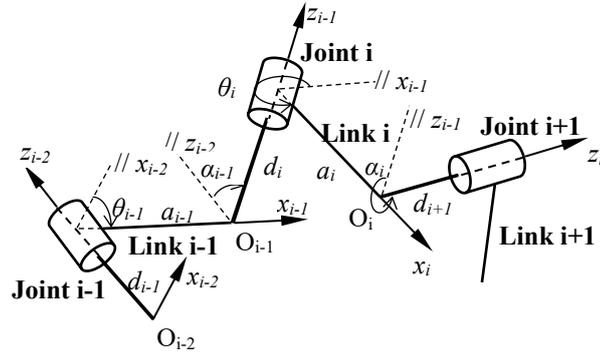
Overconstrained mechanisms have been studied by researchers, initially for kinematical point of view and in the last years from their application point of view. Many authors, like Schatz [12], Gan and Pellegrino [13], Chen [14] and more recently Ding [15] or Song [16] have presented possible applications of these mechanisms.

A new possible application of overconstrained mechanisms have been presented by Racila [17, 18], based on 6R Wohlhart symmetric mechanism, obtaining a 6R translational device. A generalization of this translational device was made it by Zeng [19], with a new design based on general forms of spatial overconstrained linkages.

The 6R translational device have three non-adjacent joints in a fixed horizontal plane, while the other three joints will be forced by the new design to remain in a plane, parallel with the horizontal plane

## 2. The new spatial disposition

In its classical position (figure 2a), the Bricard rectangular mechanism is analyzed with the help of Denavit-Hartenberg parameters. In figure 1 is shown three revolute joints using Denavit and Hartenberg notations, where  $\theta_i$  is the kinematical variable and  $a_i$ ,  $\alpha_i$  and  $d_i$  are geometrics parameters.



**Figure 1.** Denavit – Hartenberg parameters

The condition to obtain a closed loop mechanism is that the six transfer matrix product to be equal with the unity matrix:

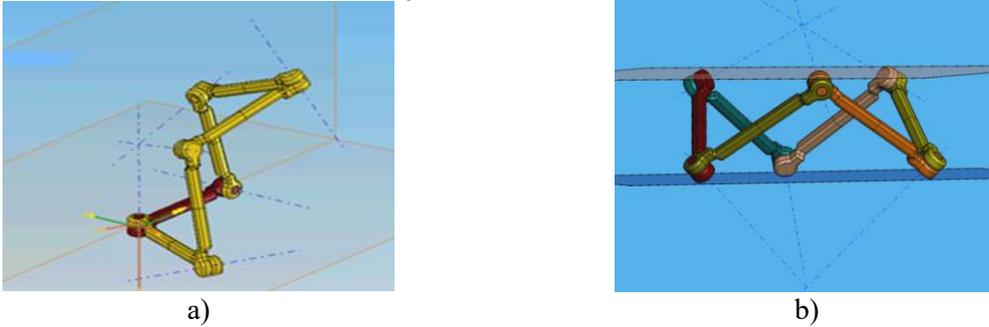
$${}^6Q_1 \cdot {}^1Q_2 \cdot {}^2Q_3 \cdot {}^3Q_4 \cdot {}^4Q_5 \cdot {}^5Q_6 = I \quad (1)$$

Using D-H parameters, the input - output equation between the input angle  $\theta$  and the output angle  $\phi$  is [17]:

$$\cos \theta \cdot \cos \phi \cdot (1 + \cos^2 \alpha) + (\cos \theta + \cos \phi) \cdot \sin^2 \alpha - 2 \cdot \sin \theta \cdot \sin \phi \cdot \cos \alpha + \cos^2 \alpha = 0 \quad (2)$$

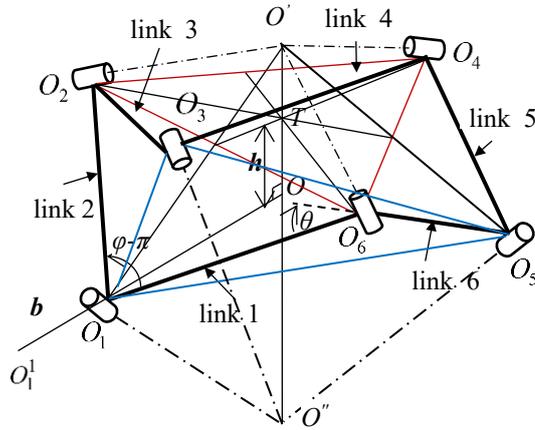
In equation (2)  $\alpha$  is the twist angle.

In the new spatial disposition (figure 2b), three non-adjacent joints are constrained to have movements in a horizontal plane to the centre of their circumscribed circle. The horizontal plane is supposed to be a fixed one. The others three joints will describe a plane, parallel with the horizontal one.



**Figure 2.** The 6R mechanism in classical position (a) and the new spatial disposition (b)

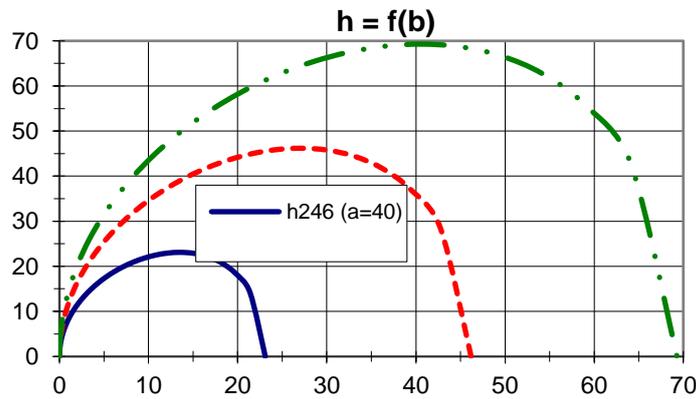
The new design for the mechanism is considering (figure 3) and in the initial position all the six joints are in the horizontal plane and the input angle  $\theta$  has zero degrees. The actuated joint is the revolute joint  $O_1$ , situated in the initial position in  $O_1^1$ . When  $O_1$  is actuated, all the odd joints  $O_1$ ,  $O_3$  and  $O_5$  make movements to the centre  $O$  of circumscribed circle of the triangle  $O_1O_3O_5$ . The even joints  $O_2$ ,  $O_4$  and  $O_6$  will be situated in a parallel plane with the horizontal and fixed plane. The distance between the two planes is noted  $h$  and this distance depend to the input angle  $\theta$  and the input distance  $b$ .



**Figure 3.** The the new spatial disposition in an intermediary position

This variation of the distance  $h$  between the horizontal plane (135) and the mobile plane (246) according to the input displacement  $b$  is [18]:

$$h = a \cdot \sqrt{\frac{4}{3} \cdot \left(1 - \frac{(2a - b\sqrt{3})^2}{4a^2}\right) \cdot \left(1 - \frac{a^2}{(2a - b\sqrt{3})^2}\right)} \quad (3)$$



**Figure 4.** The variation of the height  $h$  according to the input distance  $b$

This variation of the distance  $h$  between the horizontal plane and the mobile one, according to the input displacement  $b$  is given in figure 4 for three links length.

### 3. Focusing table

Starting from this new design, an interesting device can be obtained. The cross point of even joints axes noted  $O'$  and the cross point of odd joints axes, noted  $O''$ , form a straight line  $O'O''$  always orthogonal to the two planes, the horizontal fixed plane and the mobile one.

For a twist angle  $\alpha = \pi/2$ , the initial position is defined by the all six links in the horizontal plane. In this position the point  $O'$  is to infinity because the axes of the odd joints are parallel. The  $O''$  point is situated in the same horizontal plane, with all six revolute joints. When the input parameter  $b$  increases the point  $O'$  will move to the horizontal plane and the point  $O''$  will move under the horizontal plane, arriving finally to infinity, when all six joints will be in the horizontal plane, but in a different position.

This property can be utilized in a opposite direction, when the input movement is the displacement of the cross point  $O''$  and the output parameter is the distance between the horizontal (135) and the mobile plane (246).

To obtain this dependence, we start from the input-output equation of Bricard rectangular mechanism, with a twist angle  $\alpha=\pi/2$ , a particular case of equation (2):

$$\cos \theta + \cos \phi + \cos \theta \cdot \cos \phi = 0 \quad (4)$$

We note by  $z$  the distance  $OO''$ . From figure 3 it can be written:

$$(OO'')^2 = z^2 = a^2 \cdot \cos^2 \frac{\theta}{2} \cdot \left( \frac{1}{\sin^2 \frac{\theta}{2}} - \frac{4}{3} \right)^2 \quad (5)$$

and finally:

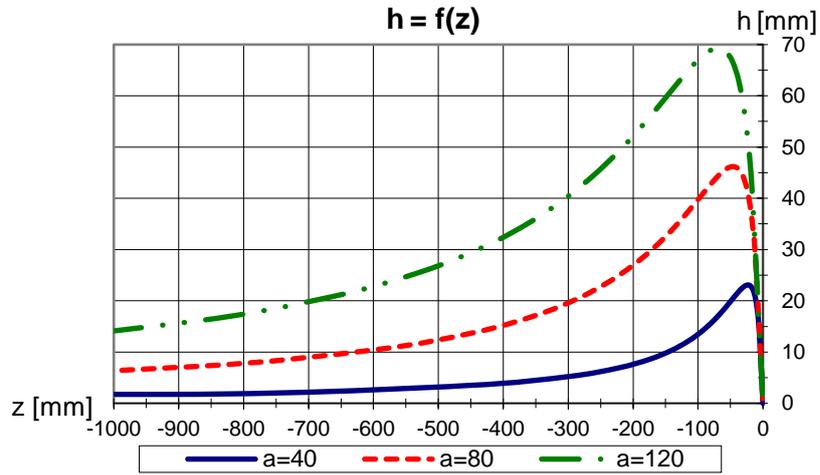
$$\cos^2 \frac{\theta}{2} = \frac{a^2 - 3 \cdot z^2 + \sqrt{(a^2 - 3 \cdot z^2)^2 + 48 \cdot a^2 \cdot z^2}}{8 \cdot a^2} \quad (6)$$

We note  $m(z)$  the expression:

$$m(z) = a^2 - 3 \cdot z^2 + \sqrt{(a^2 - 3 \cdot z^2)^2 + 48 \cdot a^2 \cdot z^2} \quad (7)$$

The expression of the height  $h$  between the two planes according to the distance  $z$  is:

$$h = \sqrt{\frac{[m(z) - 2 \cdot a^2] \cdot [8 \cdot a^2 - m(z)]}{6 \cdot m(z)}} \quad (8)$$



**Figure 5.** Variation of the height  $h$  according to  $z$  for three links length

In figure 5 is shown the variation of the distance  $OO''$ , noted  $h$ , according to the input distance  $z$ , for three links length. A similar analyze can be obtained for different twist a variation twist angle, for example  $\alpha=2\pi/3$  and  $\alpha=5\pi/6$ .

A virtual model of this device has been realized, confirming the accuracy of the analytical calculus. This accuracy can vary from 1:100 to 1:1000, depending of the links length and  $O''O$  variation domain.



**Figure 6.** Virtual model of focusing device

#### 4. Conclusion

The paper presents a 6R overconstrained closed-loop mechanism, not in its classical position, with one fixed element, but in a new spatial disposition, with no fixed elements. This special spatial position is used to obtain a new device, with three non adjacent joints imposed to make a particular movement in a fixed plane.

This device can be used to develop many applications, in the industrial area, or even in the entertainment domain. With the actuator placed in the cross point of the three nonadjacent joints of fixed plane, this device can be used, for example, to realize a focusing table for optical microscopes, or as plate table in rectifying machines, due to the its accuracy in this particular position. Some other devices are can be imagined, depending on the domain in which they are intended to be used.

#### References

- [1] Sarrus P F 1853 Note sur la transformation des mouvements rectilignes alternatifs, en mouvements circulaires et réciproquement, *Comptes Rendus des Séances de l'Académie des Sciences*, vol 36, p 1036-1038
- [2] Bricard R . 1897 Mémoire sur la théorie de l'octaèdre articulé, *Journal de Mathématiques pures et appliquées*, vol 3, p 113–148
- [3] Bricard R. 1927 *Leçons de Cinématique, Tome II – Cinématique Appliquée*, ed Gauthier (Villars, Paris)
- [4] Baker J E 1980 An Analysis of the Bricard Linkages, *Mechanism and Machine Theory*, vol 15, p. 267–286
- [5] Wohlhart K 1987A New 6R Space Mechanism, *Proc. of the 7<sup>th</sup> IFToMM World Congress*, (Sevilla), p 193–198
- [6] Bennett G T 1903 A new Mechanism, *Engineering* vol 76, p 777–778
- [7] Myard F E 1931 Chaîne fermée à cinq couples rotoïdes, déformable au premier degré de liberté, *Comptes Rendus des Séances de l'Académie des Sciences*, vol 192, p 1352-1354
- [8] Goldberg M 1943 New five–bar and six–bar linkages in three dimensions, *Transaction of the A.S.M.E.*, vol 65, p 649–663
- [9] Waldron K J 1979 Overconstrained Linkages, *Environment and Planning B*, vol 6, p 393-402
- [10] Mavroidis C and Roth B 1994 Analysis and Synthesis of Overconstrained Mechanisms, *Proc. of the ASME Design Technical Conferences*, vol. DE-70, p 115-133 (Minneapolis)
- [11] Dietmaier P 1995 A new 6R space mechanism, *Proc of the 9th IFToMM World Congress*, (Milano) p. 52–56
- [12] Schatz P 1975 *Rhythmusforschung und Technik* Ed Verlag Freies Geistesleben (Stuttgart)
- [13] Gan W W and Pellegrino S 2003 Closed-loop deployable structures, *Proc. of 44th AIAA Structures, Structural Dynamics, and Materials Conference*, (Norfolk)
- [14] Chen Y, You Z and Tarnai T 2005 Threefold-symmetric Bricard Linkages for Deployable Structures, *International Journal of Solids and Structures*, vol 42 p 2287-2301
- [15] Ding W, Qiang R, Yao Y 2016 Design and Locomotion Analysis of a Novel Deformable Mobile Robot with Two Spatial Reconfigurable platforms and Three Kinematic Chains *Proc. of the Institution of Mechanical Engineers, Part C Journal of Mechanical Engineering Science*, vol 231 issue 8, p 1481-1499
- [16] Song X, Deng Z, Guo H, Liu R, Li L and Liu R 2017 Networking of Bennett linkages and its application on deployable parabolic cylindrical antenna, *Mechanism and Machine Theory*, vol 109, p 95–125
- [17] Racila L and Dahan M 2010 Spatial properties of Wohlhart symmetric mechanism, *Meccanica*, vol 45, p 153-165
- [18] Racila L and Dahan M 2011 6R Parallel Translational Device *Proc. of the 13th IFToMM World Congress*, (Guanajuato)
- [19] Zeng Q and Ehmann K F 2014 Design of parallel hybrid-loop manipulators with kinematotropic property and deployability, *Mechanism and Machine Theory*, vol. 71, p 1-26